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COMPARISON AND SELECTION OF OBJECTIVE FUNCTIONS IN MULTIOBJECTIVE COMMUNITY DETECTION

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Detecting communities of complex networks has been an effective way to identify substructures that could correspond to important functions. Conventional approaches usually consider community detection as a single-objective optimization problem, which may confine the solution to a particular community structure property. Recently, a new community detection paradigm is emerging: multiobjective optimization for community detection, which means simultaneously optimizing multiple criteria and obtaining a set of community partitions. The new paradigm has shown its advantages. However, an important issue is still open: what type of objectives should be optimized to improve the performance of multiobjective community detection? To exploit this issue, we first proposed a general multiobjective community detection solution (called NSGA-Net) and then analyzed the structural characteristics of communities identified by a variety of objective functions that have been used or can potentially be used for community detection. After that, we exploited correlation relations (i.e., positively correlated, independent, or negatively correlated) between any two objective functions. Extensive experiments on both artificial and real networks demonstrate that NSGA-Net optimizing over a pair of negatively correlated objectives usually leads to better performances compared with the single-objective algorithm optimizing over either of the original objectives, or even to other well-established community detection approaches.

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1. INTRODUCTION

In the past decade, the complex network has attracted a large amount of researchers from different fields, because the ubiquitous complex systems in real world can be represented as networks. In complex networks, nodes denote objects in system, and edges denote their interactions. The complex network has many important characteristics, such as "small world" and long-tailed distribution. Community structure is another important characteristic of complex networks. Generally, communities are groups of nodes that are densely interconnected but only sparely connected with the rest of the network (Girvan1 and Newman 2002). Many phenomena show that community structure plays important roles in complex systems. Thus, detecting communities can acquaint us with important functions and internal structure of complex systems (Flake et al. 2002).

Many community detection algorithms have been proposed (Newman and Girvan 2004; Pizzuti 2008; Shi et al. 2010b). In a general community detection process, one single objective function is designed to capture the intuition of a community, and then it is optimized to reach an optimal value. Because optimizing these objective functions is usually an NP-hard problem, many approximation methods are employed to obtain local optimal solutions, such as spectral method (Pothen, Sinmon, and Liou 1990) and genetic algorithm (Pizzuti 2008; Shi et al. 2010a). Therefore, we can define the community detection problem

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(Ω, O) as a Single-objective Optimization Problem (SOP) (Shi et al. 2012): determine the partition C^* for which

$$O(C^*) = \min_{C \in \Omega} O(C) \tag{1}$$

where Ω is the set of feasible partitions, C is a community structure of a given network G, and $O : \Omega \rightarrow \mathcal{R}$ is an objective function. Here, we suppose that the objective O is to be minimized. The single-objective optimization paradigm is widely used in the community detection field. For example, the modularity optimization methods (Guimera and Amaral 2005; Newman and Girvan 2004; Pizzuti 2008; Shi et al. 2010b) optimize the modularity Q (Newman and Girvan 2004); the spectral clustering method optimizes the "cut" function (Luxburg 2007). These algorithms have been successively applied to many artificial and real networks. However, they also have some disadvantages (Shi 2012). For example, optimizing of just one objective function may lead to bias on community structure, and one fixed community partition returned by the single-objective algorithms may not reveal the complex community structure.

It might be a more natural and reasonable way to evaluate the community structure from different perspectives at the same time. In other words, multiple objective functions are employed to simultaneously capture the intuition of a community. This paradigm helps to avoid the structure predilection existing in single-objective approaches. Moreover, it comprehensively considers community structure information from different aspects, which may lead to a more accurate community structure. As a consequence, the community detection can be formulated as a multiobjective optimization problem (MOP). That is, the community detection problem corresponds to discover community structures that are optimal on multiple objective functions, instead of one single objective function in those single-objective approaches. Recently, some multiobjective optimization algorithms for community detection have been proposed (Agrawal 2011; Folino and Pizzuti 2010; Pizzuti 2009; Shi 2012), which have shown their advantages in generating a set of solutions and recommending more meaningful solutions.

However, for this new community detection paradigm, some important issues are still unsolved. Many optimization objectives have been proposed to capture the intuition of communities from different perspectives (Section 4.1). The communities identified by these optimization objectives have different characteristics, and these objectives have internal correlations. How do the optimization objectives affect the performance of multiobjective community detection? What type of objective functions should be optimized to improve the accuracy of community partition? The answer of these questions not only provides substantial insight to multiobjective community detection but also can explain the difference between the single-objective and multiobjective community detection. In addition, the answer can guide the design of multiobjective optimization learning algorithms.

To solve these issues, we first propose a general multiobjective community detection solution, called NSGA-Net, which can optimize over any objective functions. Then, we study the structural characteristics of communities identified by 11 popular objective functions and propose the concept of objective correlation to divide the relations between any two objective functions into three categories: positively correlated, independent, and negatively correlated. Finally, we compare NSGA-Net optimizing over six pairs of objective functions from these three types of correlations (two pairs for each type) to an SOP-based approach optimizing over the original single objective. With extensive experiments on both artificial and real networks, we find the following interesting phenomena. (1) The communities identified by different objectives have different structural characteristics.

(2) These objectives have intrinsic correlations, which determine their different behaviors. (3) A more startling finding is that NSGA-Net only with negatively correlated objectives usually leads to a better performance than that can be achieved by any of the original objectives. We also show that, with a pair of negatively correlated objectives, the NSGA-Net performs better than most conventional community detection algorithms. These findings not only provide a user with guidance in choosing the most suitable objective functions in the context of networks and applications but also benefit for the design of multiobjective community detection algorithms.

The rest of the paper is organized as follows. Section 2 formulates the multiobjective community detection problem, and we propose a general solution method in Section 3. Section 4 summarizes the characteristics of communities identified by 11 objective functions and analyzes their intrinsic correlations. Then, we compare and discuss the performance of the multiobjective community detection method with different type of objectives in Section 5. We compare the related work in Section 6. Finally, Section 7 concludes this paper.

2. PROBLEM DEFINITION

The concept of community is a generalization of human's perception: densely interconnected but sparely connected with the rest of the network. Although many criteria (i.e., objective functions) have been proposed to evaluate the quality of a community partition from different angles (e.g., modularity Q (Newman and Girvan 2004) and "cut" function (Shi and Malik 2000)), the intuition of human is difficult to be captured by one single criterion. Thus, it is a natural way to treat the community detection as a MOP. That is, in the multiobjective community detection problem $(\Omega, O_1, O_2, \ldots, O_t)$ (Shi et al. 2012), we aim to discover the community structure C^* for which

$$O(C^*) = \min_{C \in \Omega}(O_1(C), O_2(C), \ldots, O_t(C)) \tag{2}$$

where t is the number of objectives and O_i represents the ith objective. There is usually no single best solution for an MOP. Here, the notion of dominance (Deb 2001) is introduced. For two partitions $C_1, C_2 \in \Omega$, the partition C_1 is said to dominate the partition C_2 (denoted as $C_1 \preceq C_2$) if and only if

$$\forall i \in \{1, \ldots, t\}\ O_i(C_1) \le O_i(C_2) \wedge \exists i \in \{1, \ldots, t\}\ O_i(C_1) < O_i(C_2) \tag{3}$$

Another important notation in multiobjective optimization is Pareto optimal (Deb 2001). A partition $C \in \Omega$ is said to be Pareto optimal if and only if there is no other partition dominating C. The set of all Pareto optimal partitions is the Pareto optimal set and the corresponding set in the objective space is the nondominated set, or Pareto front (Deb 2001).

Compared with single-objective approaches, the multiobjective community detection has many advantages in theory, which has been detailedly analyzed in reference Shi et al. (2012). First, the Pareto optimal set of the multiobjective problem (i.e., $(\Omega, O_1, \ldots, O_t)$) always comprises the optimal solutions of the single-objective community detection problem (i.e., $(\Omega, O_1), \ldots, (\Omega, O_t)$). Moreover, the multiple objectives help to avoid the risk that one single objective may only be suitable to a certain kind of network. In addition, a set of community partitions returned by the multiobjective community detection contribute to discover complex and comprehensive community structures.

For this multiobjective community detection problem, several solutions have been proposed (Agrawal 2011; Folino and Pizzuti 2010; Pizzuti 2009; Shi et al. 2012). These algorithms usually simultaneously optimize two objective functions with a random search method (e.g., evolutionary algorithm (EA)) and return a set of partitions. Experiments also proved that these algorithms can recommend more meaningful partitions. However, a common and important issue in these algorithms has seldom been explored: what type of objective functions should be optimized in this multiobjective community detection paradigm? It is not a trivial problem. As we known, the optimization objectives guide the search process of algorithms, which greatly determines the algorithm performance. On the other hand, researchers have proposed many objective functions to evaluate communities from different perspectives (Section 4.1). When one designs a multiobjective community detection algorithm, it is hard to choose the optimization objectives. Thus, we should study the characteristics and correlations of objective functions and explore the effect of the combination of different objectives on performances. Although this problem is very important, it is seldom explored as far as we know. In the multiobjective optimization field, the researchers consciously choose the conflicting objectives (Deb 2001; Handle and Knowles 2007; Pizzuti 2009), but no one validates this point in a universal form and even explains any reason insight. In this paper, we try to solve this problem by studying the general performance of the multiobjective community detection algorithm under the combination of different objectives. This study not only benefits the design of multiobjective community detection algorithm but also helps to make a thorough inquiry to the new paradigm.

3. THE NSGA-NET SOLUTION

Although a number of algorithms (Agrawal 2011; Folino and Pizzuti 2010; Pizzuti 2009; Shi et al. 2012) have been proposed to solve the multiobjective community detection problem, they are restricted to concrete optimization objectives. In this paper, we propose a general multiobjective community detection solution (called NSGA-Net), which can optimize any objective functions. NSGA-Net is based on EA. EA has been proven to be an effective method for MOP. Evolutionary multiobjective optimization (EMO) is not only an effective solution for MOP but also shows its potential in data mining (Handle and Knowles 2007; Shi et al. 2011a). Conventional EMO algorithms are designed to solve numerical optimization problems. We need to redesign many components of EA for a real problem. It is not a trivial task because the algorithm performance is determined by the design of these components to a large extent. The NSGA-Net includes two phases: (1) the community detection phase that simultaneously optimizes multiple objectives and returns a set of community partitions and (2) the model selection phase that selects the most preferable solution from the partition set.

3.1. Community Detection Phase

In this phase, NSGA-Net employs the NSGA-II (Deb et al. 2002) as the multiobjective optimization framework. The *locus-based adjacency* coding schema is applied as the genetic representation and corresponding operators are designed.

Multiobjective optimization mechanism. In this paper, we select NSGA-II (Deb et al. 2002) as the multiobjective optimization mechanism in NSGA-Net, because NSGA-II has been proven to be an effective and efficient EMO in numerical optimization. The basic idea in NSGA-II is to transform the optimized objectives to a fitness measure by the creation of a number of fronts with a density estimation. In each generation, the strategy of survival of the fittest is performed, and thus, an elite set can be kept from generation to generation.

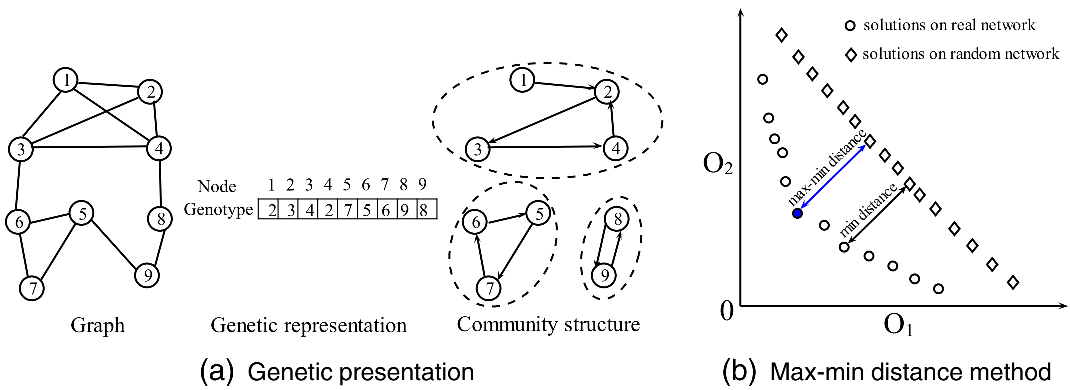


FIGURE 1. (a) Genetic representation and its corresponding community structure. (b) Illustration of the *Max–Min Distance* model selection method.

Four parameters govern the run of NSGA-Net: the population size *popSize*, the running generation *gen*, the ratio of crossover *croRat*, and the ratio of mutation *mutRat*.

Genetic representation. In EA, a genetic representation should be employed to encode a community partition with a character string (i.e., genotype). Inversely, the genotype can also be decoded into a community partition. We apply the *locus-based adjacency* representation (Park and Song 1989) in NSGA-Net. An example is illustrated in Figure 1(a). For a network with size n, a genotype g consists of n genes $< g_1, g_2, \ldots, g_n >$, and each g_i is one of the adjacent nodes of node i. Thus, the ith gene assigning with j (i.e., $g_i = j$) means that there is a link between node i and j. In the partition, they will be in the same community. For example, in Figure 1(a), gene g_2 is 3; thus, node 2 links to node 3, and they are in the same community. The decoding process needs to identify all connected components. All the nodes in the same connected component belong to one community. The *locus-based adjacency* representation has shown its superiority in community detection (Shi et al. 2010b, 2012).

Genetic operators. NSGA-Net applies the uniform two-point crossover, because this operator is able to generate any combination of two parent genotypes. Thus, the crossover operator guarantees that no invalid solutions will be generated. The mutation operation randomly selects some genes and assigns them with other randomly selected adjacent nodes.

Initialization. The initialization process randomly generates a set of individuals. For each individual (i.e., genotype), each gene g_i randomly takes one of the adjacent nodes of node i.

3.2. Model Selection Phase

NSGA-Net returns a set of solutions, which provides decision makers with more choices. However, sometimes decision makers may desire to narrow the candidate solutions down to those of most interest. This paper applies the *Max–Min Distance* model section method (Shi et al. 2012) to select one single recommendation solution from the Pareto front. It also aims to conveniently compare NSGA-Net with conventional single-objective algorithms, because those algorithms only return one single solution. The *Max–Min Distance* method selects the solution model that mostly deviates from the null models generated by NSGA-Net by running on random networks with the same scale. That is, the random network has the same number of nodes and edges with the real network. Thus, the optimal solution set on the real network (called *CandSet*) and the corresponding random network (called *RandSet*) can be obtained, respectively. For each solution in *CandSet*, we calculate the minimum distance with solutions in *RandSet*, and then we select the solution in *CandSet* with the maximum–minimum distance as the recommendation solution. Here, Euclidean distance is employed. As an example shown in Figure 1(b), the solution marked with filled dot is the recommendation solution. Intuitively, the recommendation solution is the most distinct one from the solutions in *RandSet*. The *RandSet* aims to estimate the expected objective values for unstructured networks. The *Max–Min Distance* method find the solution that most deviates from randomness, which means the recommendation solution has the most obvious community structure.

3.3. Objective Functions

Still NSGA-Net has an important component unsolved: optimization objectives. As a general multiobjective community detection solution, NSGA-Net can employ any objective functions. With NSGA-Net, we will examine the general performance of the multiobjective community detection. Furthermore, we explore what type of objectives is suitable for the multiobjective paradigm. To do so, we first make a deliberate investigation on the characteristics of these objectives and their intrinsic correlations.

4. OBJECTIVE FUNCTIONS AND THEIR CORRELATIONS

This section analyzes the structure characteristics of communities identified by popular objective functions and the intrinsic correlations among these objectives.

4.1. Objective Functions

Many objective functions have been proposed to capture the intuition of communities. We summarize 11 objective functions that are already widely used in community detection literatures or can be potentially used for community detection (Shi et al. 2010a, 2011b). Let $G(V, E)$ be an undirected graph with $n = |V|$ nodes and $m = |E|$ edges. Let C be a partition with l communities and S be the set of nodes in one community, where $C = \{S_1, S_2, \ldots, S_l\}$. n_S is the number of nodes in S, $n_S = |S|$, m_S is the number of edges in S, $m_S = |(u, v) \in E : u \in S, v \in S|$, and c_S is the number of edges on the boundary of S, $c_S = |(u, v) \in E : u \in S, v \notin S|$; and $d(u)$ is the degree of node u. The objective $O(C)$ is the quality of a partition C.

- Conductance: $O_1(C) = \sum_{S \in C} \frac{c_S}{2m_S + c_S}$ measures the fraction of total edge volume that points outside the cluster (Kannan, Vempala, and Vetta 2004).
- Expansion: $O_2(C) = \sum_{S \in C} \frac{c_S}{n_S}$ measures the number of edges per node that point outside the cluster (Radicchi et al. 2004).
- Cut ratio: $O_3(C) = \sum_{S \in C} \frac{c_S}{n_S(n - n_S)}$ is the fraction of all possible edges leaving the cluster (Fortunato 2009).
- Normalized cut: $O_4(C) = \sum_{S \in C} \left(\frac{c_S}{2m_S + c_S} + \frac{c_S}{2(m - m_S) + c_S} \right)$ is the normalized fraction of edges leaving the cluster (Shi and Malik 2000).
- Maximum out degree fraction (ODF): $O_5(C) = \sum_{S \in C} \max_{u \in S} \frac{|\{(u,v) : v \notin S\}|}{d(u)}$ is the maximum fraction of edges of a node pointing outside the cluster (Flake, Lawrence, and Giles 2000).
- Average-ODF: $O_6(C) = \sum_{S \in C} \frac{1}{n_S} \sum_{u \in S} \frac{|\{(u,v) : v \notin S\}|}{d(u)}$ is the average fraction nodes' edges pointing outside the cluster (Flake et al. 2000).

- Flake-ODF: $O_7(C) = \sum_{S \in C} \frac{|\{u:u\in S,|\{(u,v):v\in S\}|<d(u)/2\}|}{n_S}$ is the fraction of nodes in S that have fewer edges pointing inside than to the outside of the cluster (Flake et al. 2000).
- Q: $O_8(C) = \sum_{S \in C} \left(\frac{m_S}{m} - \left(\frac{m_S + c_S}{2m}\right)^2 \right)$ measures the number of within-community edges, relative to a null model of a random graph with the same degree distribution (Newman and Girvan 2004).
- Description length: $O_9(C) = n \log l + \frac{1}{2} l(l+1) \log m + \log \left(\prod_{i=1}^{l} \binom{n_i(n_i-1)/2}{m_i} \prod_{i>j} \binom{n_i n_j}{c_{ij}} \right)$ where n_i and m_i are the number of nodes and edges in community i, respectively; c_{ij} is the number of edges between the community i and j. The objective regards the community as an optimal compression of the network's topology (Martin and Carl 2007).
- Community score: $O_{10}(C) = \sum_{S \in C} (2m_S/n_S)^2$ measures the density of a sub-matrices based on volume and row/column means (the power order r is 1 for simplicity) (Pizzuti 2008).
- Internal density: $O_{11}(C) = \sum_{S \in C} \left(1 - \frac{m_S}{n_S(n_S-1)/2}\right)$ is the internal edge density of the cluster (Radicchi et al. 2004).

We roughly classify these objective functions into three categories. The first category contains the first four objectives (i.e., *Conductance*, *Expansion*, *CutRatio*, and *NormalizedCut*) from graph theory community. Because they all consider the "cut" in a graph, we call them the cut-based objectives. The three objectives ended with "ODF" (i.e., *Maximum-ODF*, *Average-ODF*, and *Flake-ODF*) all consider the degree of nodes in a community, and thus we call them degree-based objectives. Finally, the remaining objectives are classified into one category. These objective functions come from different research fields, such as graph theory and physics. All these objectives attempt to capture a group of nodes with better internal connectivity than external connectivity, and thus they all can be potentially used in community detection. Moreover, some objective functions are not considered, for example, the Hamiltonian-based method (Reichardt and Bornholdt 2006) and a multiple resolution procedure (Arenas, Fernandez, and Gomez 2008). The objective functions in both of the two methods require tuning parameters. Because the parameters are hard to choose in applications, we did not include them in this paper. Note that some objectives need to be maximized (e.g., *Q* and *CommunityScore*). To handle all the objectives in a uniform form, we convert these objectives into a minimum problem for convenience. This conversion does not affect the partition result.

4.2. Characteristics of Objective Functions

For the SOP, $\min_{C \in \Omega} O(C)$, we can optimize it with many techniques, such as spectral method, simulated annealing, and genetic algorithm. For a fair comparison, we use the same optimizer for testing all the objective functions. Particularly, we choose GACD (Shi et al. 2010b) as the single-objective community detection optimizer based on the following reasons: (1) GACD is also an EA-based community detection algorithm. The same algorithm paradigm between NSGA-Net and GACD (i.e., both EA-based algorithms) makes their differences in performances mainly attributed to the multiobjective optimization in NSGA-Net. (2) GACD is a general single-objective community detection optimizer, which can optimize any objective function. (3) GACD has been proven as an effective community detection algorithm (Shi et al. 2010b).

4.2.1. Experiments on Artificial Networks. We use a popular artificial network with a known community structure (Lancichinetti, Fortunato, and Radicchi 2008). The network has the heterogeneity in the distributions of node degrees and community sizes, which is widely used in many research (Gong et al. 2011; Shi et al. 2012). As suggested in reference Lancichinetti et al. (2008), the benchmark graphs are as follows: the number of nodes is $N = 1500$; the average degree is $k = 25$ and the maximum degree is 80; the degree and the community size distributions are power laws, with exponents $\gamma = 2$ and $\beta = 2$, respectively. The μ is the mixing parameter, which controls the fraction of a node connecting with nodes outside the community. As μ increases, it becomes more difficult to identify the community structure.

To compare the built-in modular structure with the result returned by different objectives, we adopt the normalized mutual information (NMI), which is a popular measure of similarity of partitions from information theory (Lancichinetti et al. 2008). The parameters in GACD for all objectives are set as follows: $popSize = gen = 200$, $croRat = 0.6$, and $mutRat = 0.4$ (the same parameters are set in the following experiments). The experimental results, as shown in Figure 2, are an average over 20 graph realizations. As the community structures become fuzzy (i.e., μ increases), it becomes difficult for all objectives to discover the real structures. For all networks, *CommunityScore* and Q have the highest accuracy. The cut-based objectives have the similar performance. That is, as μ increases, these objectives tend to divide the graph into two communities. It is not particularly surprising as the cut-based objectives are more approximate to the optimal value in this condition. The degree-based objectives also have the similar behavior that they combine all nodes as one community as μ increases. Its rationality is that these objectives always reach the minimum value 0 in this condition. From Figure 2(b), we can find that all objectives, except *CommunityScore*, have the resolution limit problem (Fortunato and Barthelemy 2007), because the number of communities identified by these objectives is smaller than the real number. In other words, these objectives all tend to combine some small communities into large ones.

4.2.2. Experiments on Real Networks. To study the characteristics of communities identified by different objective functions, we further optimize these objective functions with GACD on the 12 real networks shown in Table 1. These networks with medium and

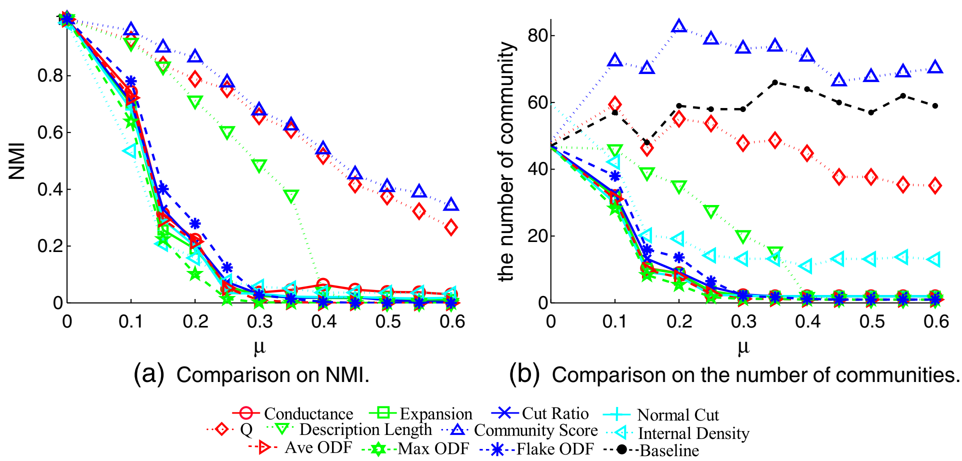
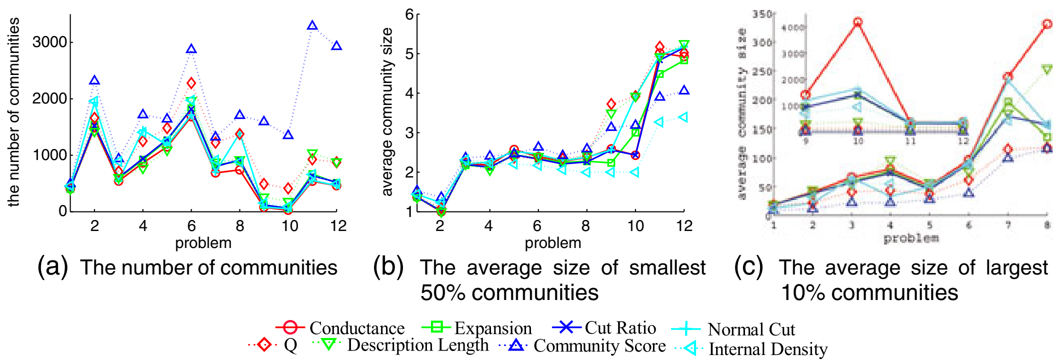


FIGURE 2. The normalized mutual information comparison of communities identified by 11 objectives on the artificial networks. The baseline shows the real number of communities.

TABLE 1. Real Networks and Their Size.

	Net-scie. (P1)	Hep-th (P2)	CA-GrQc (P3)	CA-Hep. (P4)	PGPgian. (P5)	CA-Con. (P6)
No. of nodes	1589	8361	5242	9877	10,680	23,133
No. of edges	2,742	15,751	28,980	51,971	24,316	186,936
	CA-Ast. (P7)	**Cit-Hep. (P8)**	**P2p-04 (P9)**	**P2p-06 (P10)**	**P2p-24 (P11)**	**P2p-25 (P12)**
No. of nodes	18,772	27,770	10,876	8717	26,518	22,687
No. of edges	396,160	352,807	39,994	31,525	65,369	54,705



(a) The number of communities (b) The average size of smallest 50% communities (c) The average size of largest 10% communities

FIGURE 3. The statistical analysis of the distribution of the community size on the 12 problems.

large size are from the popular common data sources (Leskovec 2010; Newman 2009). The distribution of community size of their results in all 12 problems is shown in Figure 3. The experimental results do not include those of the degree-based objectives, because we find that they all are very prone to divide the whole network into one community.

In Figure 3(a), we can observe that *CommunityScore* and Q find the maximum number of communities and the cut-based objectives reveal the minimum number of communities on most networks. We further show the average size of the smallest 50% and the largest 10% communities identified by the eight objectives on the 12 networks in Figures 3(b) and (c), respectively. It shows that most communities are very small. There are no obvious differences on the size of small communities identified by different objectives (Figure 3(b)). However, it is not the case for large communities (Figure 3(c)). For most networks, the cut-based objectives always find larger communities, and *CommunityScore* and Q have the opposite trend. Similar to the cut-based objectives, *DescriptionLength* also tends to find a small number of communities with the large size. It is interesting for *InternalDensity* that simultaneously finds many small communities and some large communities.

In all, the experiments on artificial and real networks show that *CommunityScore* and Q divide networks with a finer granularity (i.e., more communities with smaller size). The cut-based objectives, degree-based objectives, and *DescriptionLength* reveal the community structure with a coarser granularity (i.e., fewer communities with large size). The behavior of *InternalDensity* has the characteristics of both of them. The huge communities discovered by the cut-based objectives, degree-based objectives, and *DescriptionLength* may not be very meaningful, which indicates that these objectives may not be very suitable for networks with small communities. In all these experiments, the modularity Q has the stable and good performances. It might explain why Q is the most popular objective function. In addition, some objectives have very similar behavior in all these experiments, such as cut-based objectives and degree-based objectives. It shows that intrinsic correlations exist among these objective functions.

4.3. Objective Correlations

Observing the comparison results, one may ask the following questions: what causes the different performances of these objective functions? Furthermore, why do some objectives (i.e., cut-based objectives) have similar performances? We can directly observe that the definitions of some objectives are similar, such as the cut-based objectives. In other words, these objectives are correlated. Here, we apply the Pearson correlation coefficients to describe their correlations. Because it is difficult to analyze their correlations from the definitions directly, we perform experiments to estimate the Pearson correlation coefficients. The experiments are carried out with the following steps. (1) For a given network, we generate a set of random partitions. (2) For each partition, we calculate the values of the different objective functions. Thus, each objective function has a vector of random samples. (3) We estimate the Pearson correlation coefficients among these objective vectors. (4) To reduce the estimation variance, we repeat steps 1 to 3 many times and obtain the average values.

The results are illustrated in Figure 4. We can observe that the cut-based objectives are highly correlated (especially $O_1 - O_3$). It is the same case for the degree-based objectives. It explains why these objectives have so similar performances. Q (O_8) and *CommunityScore* (O_{10}) are also highly correlated, and this is the reason why they have the similar performances. In addition, we notice *InternalDensity* (O_{11}) is negatively correlated with Q and *CommunityScore*, which might lead to the opposite properties of the communities

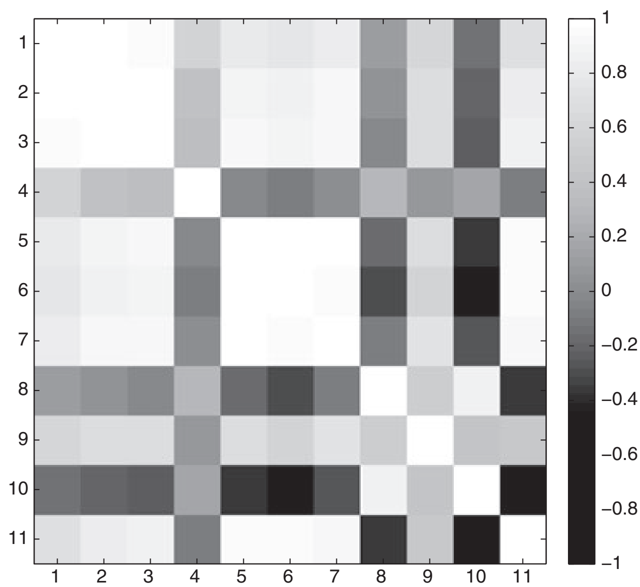


FIGURE 4. Pearson correlative coefficients of the 11 objectives. The numbers 1–11 represent the objective functions $O_1 - O_{11}$, respectively.

identified by them. The relations of these objectives can be roughly classified into three categories in terms of their correlation coefficients: positively correlated (e.g., $\{O_1, O_2, O_3\}$, $\{O_5, O_6, O_7\}$, $\{O_8, O_9, O_{10}\}$), independent (e.g., $\{O_1, O_8\}$, $\{O_1, O_{10}\}$, $\{O_4, O_9\}$), and negatively correlated (e.g., $\{O_8, O_{11}\}$, $\{O_{10}, O_{11}\}$).

5. PERFORMANCES AND OBJECTIVE SELECTION OF MULTIOBJECTIVE COMMUNITY DETECTION

In this section, we will test the performances of the multiobjective community detection method (i.e., NSGA-Net) and find what kinds of objectives are suitable for the method. Here, we only consider two objectives, rather than more objectives, to focus on the effectiveness of the multiobjective method and reduce the complexity. From each of the three categories of objective correlations, we select two pairs as the optimized objectives in NSGA-Net. Particularly, for the positively correlated objectives we choose $\{O_1, O_2\}$ and $\{O_8, O_9\}$; the independent objectives, $\{O_1, O_8\}$ and $\{O_4, O_9\}$; and the negatively correlated objectives, $\{O_8, O_{11}\}$ and $\{O_{10}, O_{11}\}$. The same networks are used as last section. NSGA-Net is equipped with the same parameters with GACD for a fair comparison.

5.1. Experimental Results on Artificial Networks

We first run NSGA-Net on artificial networks. The comparison results of NSGA-Net optimizing over six pairs of objectives and GACD optimizing over original single objectives are shown in Figure 5. When the optimized objectives are positively correlated (Figures 5(a)

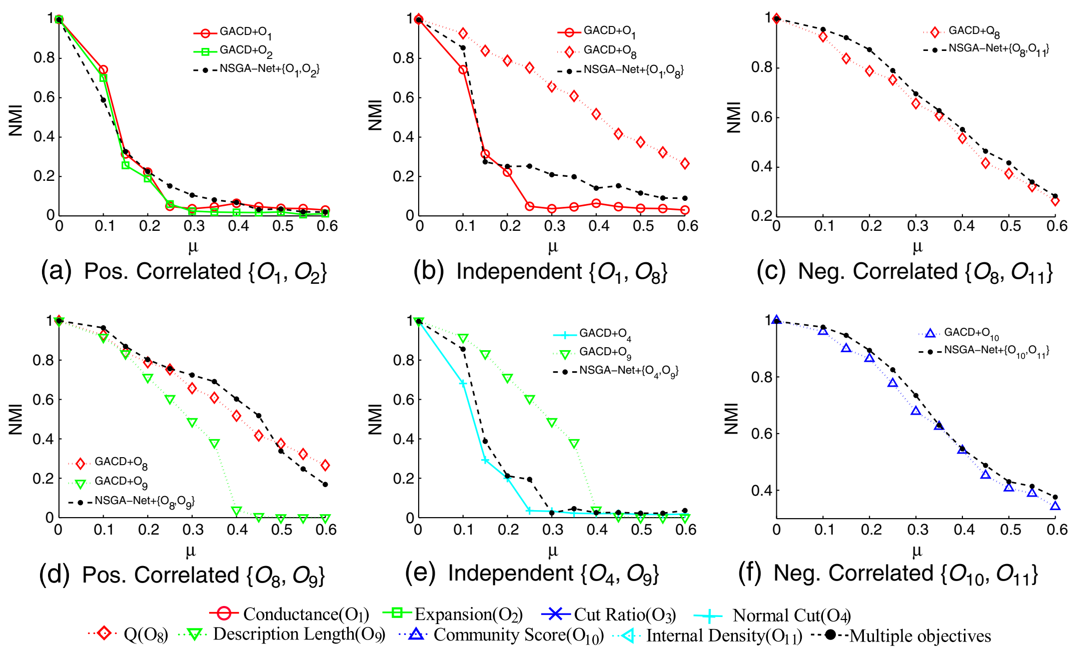


FIGURE 5. The normalized mutual information comparison of NSGA-Net optimizing over three types of objective functions (i.e., positively correlated, independent, negatively correlated) and GACD optimizing over original single objectives on artificial networks. To strengthen the difference in (c) and (f), we omit the result of $GACD + O_{11}$ that has a bad performance (Figure 2(a)). The larger the normalized mutual information, the better the performance.

and (d)) or independent (Figures 5(b) and (e)), NSGA-Net's performances have no obvious differences from the performances of the optimization on each single objective with GACD. Most results of NSGA-Net are between those of the single objectives. However, it is obvious that NSGA-Net with a pair of negatively correlated objectives (Figures 5(c) and (f)) has better performance than the optimization on the original single objective. NSGA-Net not only steadily performs better than GACD on all conditions but also improves the performance up to 6.7% and 4.7% on Figures 5(c) and (f), respectively.

5.2. Experimental Results on Real Networks

It is difficult to evaluate the quality of communities for real networks, because real community structures are unknown. Conventional criteria use an indicator to evaluate the quality of the whole partition, such as Q (Newman and Girvan 2004). However, they are not suitable for our work, becuase these criteria have bias on the optimized objectives. Moreover, these criteria can just reflect the quality of whole partition, not the quality of internal communities. Here, we propose a new criterion to evaluate the quality of communities as a function of their size, which provides a much finer resolution to examine the partition results. In detail, for a community partition $C = \{S_1, S_2, \ldots, S_l\}$, we propose the *Average Measure Function* $AMF(k)$ to evaluate the average measures of the communities with size k.

$$\begin{aligned} AMF(k) &= \frac{\sum_{S_i \in C_k} \mathrm{Crit}(S_i)}{|C_k|} \\ C_k &= \{S_i \mid S_i \in C, |S_i| = k, k \in \{1, \ldots, n\}\} \end{aligned} \tag{4}$$

where $\mathrm{Crit}(S_i)$ is a criterion that evaluate the quality of the community S_i. From the definition of $AMF(k)$, we can find that it reveals the structural characteristics of communities with size k, instead of the characteristics of all communities in conventional criteria. Note that the community size k may be not sequential. For $\mathrm{Crit}(S)$, we select two popular criteria, *Conductance* and *ShortestPath*, to evaluate the quality of a community S.

$$\begin{aligned} Conductance(S) &= c_S / \min(\mathrm{Vol}(S), \mathrm{Vol}(V \backslash S)) \\ ShortestPath(S) &= \frac{2\sum_{i,j \in S} \mathrm{dis}(i, j)}{n_s(n_s + 1)} \end{aligned} \tag{5}$$

where $\mathrm{Vol}(S) = \sum_{u \in S} d(u)$ and $\mathrm{dis}(i, j)$ is the shortest path of node i and j. Note that *Conductance* here measures the degree of connection with outsides for one community, rather than that for the whole partition as in the objective O_1. Although *Conductance* may have bias on O_1, it reflects the quality of community to some extent. A smaller *Conductance* indicates that nodes in the community are sparsely connected with outside. The *ShortestPath* is the average length of the shortest path of pairwise nodes in the community. A smaller *ShortestPath* shows that nodes in a community are closely (densely) connected with each other. Together, the two criteria reflect two aspects of a good community, that is, densely interconnected and sparsely connected with outside. As a consequence, the $AMF(k)$ function can comprehensively evaluate the quality of a community partition with a finer granularity.

Our measures (especially the *Conductance*-based measure) are similar to the Network Community Profile (*NCP*) proposed by Leskovec, Lang, and Mahoney (2010). They both are size-resolved measure, which can evaluate community structure with a finer granularity. However, they are different. First, they have different aim. *NCP* asks for approximation to the best cluster (i.e., a cluster with the smallest conductance) for every possible size.

The *AMF* evaluates the structure characteristics of communities with the same size. Second, different from the lower-envelope curves (i.e., smallest performance) of communities with size k illustrated by *NCP*, the *AMF* shows the average performance of communities with the same size. And thus, we think our measure can more comprehensively reflect the quality

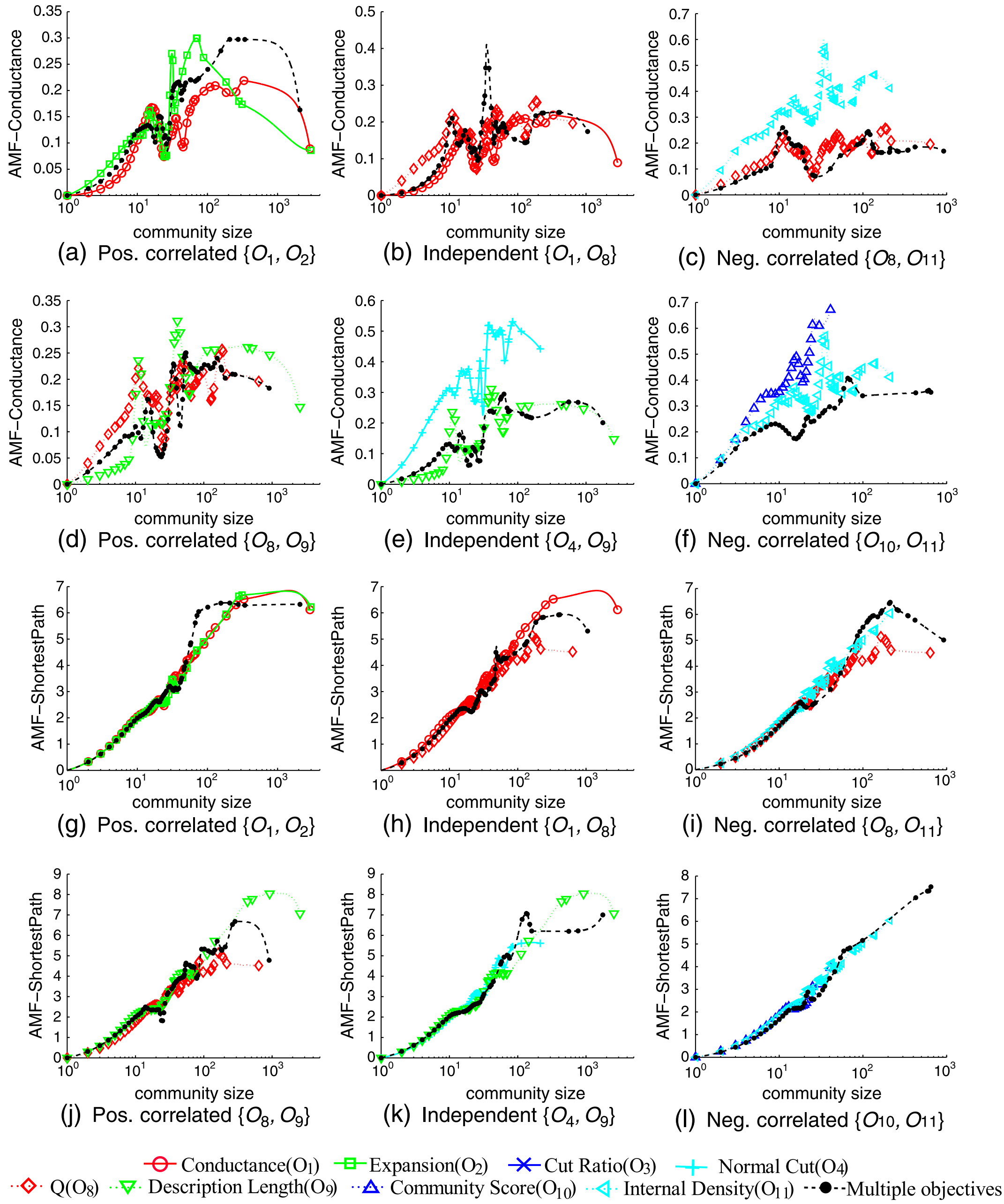


FIGURE 6. The *AMF* comparison of NSGA-Net optimizing over three types of objective functions (i.e., positively correlated, independent, and negatively correlated) and GACD optimizing over original single objectives on the P2 network. The first and last two row subgraphs are the comparing results on *AMF-Conductance* and *AMF-ShortestPath* criteria, respectively. The smaller the *AMF*, the better the performance.

of communities. Moreover, our measures are more suitable for the evaluation of algorithms used in this paper, because they both are based on clusters of any size, rather than the cluster for every possible size.

We run NSGA-Net with six pairs of objective functions and GACD with 12 corresponding objective on the 12 real networks. Then, we use the *AMF* criterion to compare the performances of NSGA-Net with that of GACD. Because of the space limitation, we only show the results on P2 network in Figure 6. Intuitively, we can observe that the *AMF* curves of NSGA-Net with negatively correlated objectives are lower than that of GACD with the original single objective in most conditions (see the last column subgraphs in Figure 6), which means NSGA-Net have better performance than GACD. However, it is not the case for NSGA-Net with the other two types of objectives, because their *AMF* curves are usually between the two *AMF* curves of GACD with original objectives (see the first two column subgraphs in Figure 6). To capture it more clearly, we define the $LowRat(C)$ to quantitatively count the ratio of communities in a partition C with the smallest measure values comparing with two other partitions. *LowRat* ranges from 0 to 1, and the larger *LowRat* means better performance.

$$LowRat(C) = \frac{\left|\left\{C_k | AMF(C_k) < AMF\left(C'_k\right)\right\}\right|}{\left|\left\{C_k | C_k \subseteq C, k \in 1, \ldots, n\right\}\right|} \tag{6}$$

where C_k and C'_k are the communities with the size k in two different partitions C and C', respectively. Taking Figure 6(f) for example, the partition result of NSGA-Net with $\{O_{10}, O_{11}\}$ (see the *AMF* curve marked with black dot) have 37 communities with different sizes, in which 21 communities have the smallest *Conductance* compared with other two partitions of GACD with O_{10} and O_{11}. And thus, its *LowRat* is 0.5676. This value is not large, although Figure 6(f) shows that the *AMF* curve of NSGA-Net with $\{O_{10}, O_{11}\}$ is overwhelmingly smaller than the other two *AMF* curves. Thus, we think the superiority is remarkable when *LowRat* is larger than 0.5. Note that there are three *AMF* curves (one for NSGA-Net with multiobjectives and two for GACD with original single objectives) in a figure, and thus the expected *LowRat* for each algorithm is $1/3$ (i.e., the baseline).

The *LowRat* values of partition results identified by NSGA-Net on 12 networks are shown in Figure 7. We find the same phenomena as that of artificial networks. The NSGA-Net with negatively correlated objectives remarkably performs better than the single-objective optimization on original single objectives, because their *LowRat* values

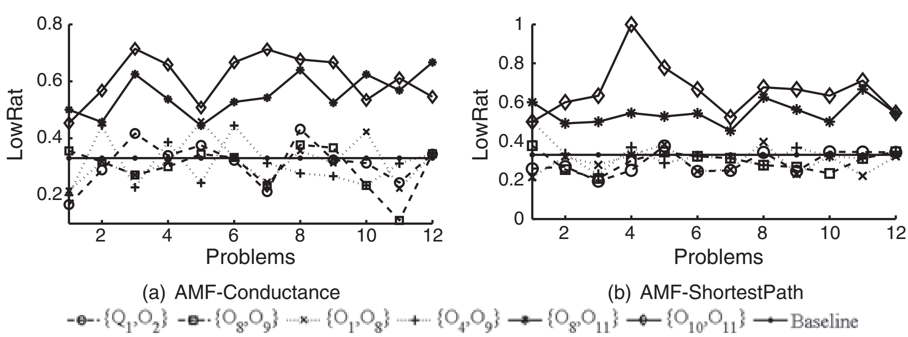


FIGURE 7. The *LowRat* values of partitions identified by NSGA-Net optimizing over three types of objective functions on 12 real networks. The larger the *LowRat*, the better the performance.

(i.e., $\{O_8, O_{11}\}$, $\{O_{10}, O_{11}\}$) are significantly larger than the baseline $1/3$ on most networks. However, it is not obvious for the NSGA-Net with other two types of objectives, because most of their *LowRat* values are around $1/3$. These experiments further confirm that the multiobjective community detection with negatively correlated objectives remarkably improves the accuracy of community partition.

5.3. Comparison with Other Algorithms

To further validate the aforementioned conclusion, we compare NSGA-Net with negatively correlated objectives to other representative community detection algorithms. NSGA-Net is equipped with a pair of negatively correlated objectives O_8 and O_{11}, because many popular algorithms optimize the O_8 (i.e., Q the facto criterion in physics field). Four well-established single-objective community detection algorithms are included in the experiments. It includes the betweenness-based heuristic algorithm (Newman and Girvan 2004) (named GN) and its improved version (Clauset, Newman, and Moore 2004) (named GN Fast). The EA-based optimization algorithm (Shi et al. 2010b) (named GACD) optimizes the O_8. The information-theoretic framework-based algorithm (named INFO) (Martin and Carl 2007) optimizes the O_9 (i.e., *DescriptionLength*). In addition, two multiobjective community detection method MOCD (Shi et al. 2012) and MOGA-Net (Pizzuti 2009) are also included. MOCD simultaneously optimizes two components of the modularity Q. MOGA-Net simultaneously optimizes the O_{10} (i.e., *CommunityScore*) and *Community-Fitness* (a criterion measuring the ratio of internal degree). To obtain one single recommendation solution, MOGA-Net and MOCD also employ the *Max–Min distance* model selection method to select a partition from the Pareto front. NSGA-Net, MOCD, and MOGA-Net

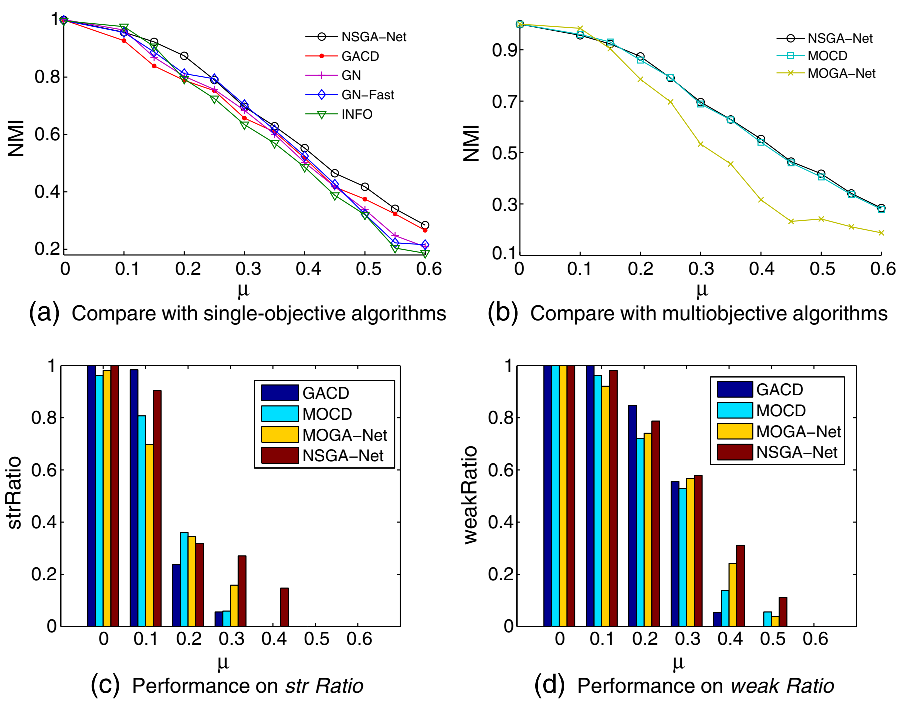


FIGURE 8. The comparison of NSGA-Net employing a pair of negatively correlated objectives (i.e., O_8 and O_{11}) with other popular algorithms on artificial networks.

are set as the same parameters with GACD in Section 4.2.1. The benchmark is the same artificial networks as before.

The experimental results are shown in Figure 8. It is clear that NSGA-Net performs better than other single-objective algorithms in most conditions, as shown in Figure 8(a). Moreover, Figure 8(b) shows that NSGA-Net has a better performance than the multiobjective method MOGA-Net. Note that MOGA-Net performs well when μ is small; however, it becomes the worst one when μ grows large. An important difference between NSGA-Net and MOGA-Net lies in the objective functions. We think that the absence of the sufficient negative correlation between objectives in MOGA-Net causes its bad performances. MOCD has a close performance to NSGA-Net. Shi et al. (2012) emphasized that two optimized objectives in MOCD are conflicting, which may explain the reason of the good performance of MOCD. We also find that the different optimization framework in NSGA-Net and MOCD (i.e., NSGA-II (Deb et al. 2002) and PESA-II (Corne et al. 2001), respectively) do not much affect the performance of multiobjective community detection.

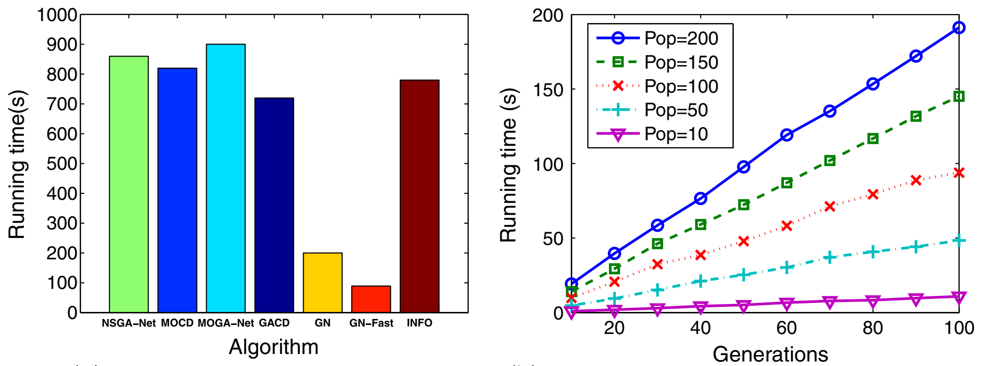
For the NMI measure, the NSGA-Net does not achieve much improvement when it is compared with other multiobjective algorithms. We further compare the structure characteristics of communities they discover through other criteria. Radicchi et al. (2004) proposed the concept of strong community and weak community to depict the closeness of a community, which is widely used in community evaluation (Costaa et al. 2007; Fortunato and Barthelemy 2007; Shi et al. 2012). Here, we validate whether each community is a strong (or weak) community and calculate the *ratio of strong (or weak) communities* to quantificationally evaluate the quality of partition. The *ratio of strong communities* (*strRatio*) and the *ratio of weak communities* (*weakRatio*) are formally defined as follows.

$$
\begin{aligned}
strRatio(C) &= \frac{\left|\left\{S | k_i^{\text{in}}(S) > k_i^{\text{out}}(S)\ \forall i \in S \wedge \forall S \in C\right\}\right|}{|C|} \\
weakRatio(C) &= \frac{\left|\left\{S | \sum_{i \in S} k_i^{\text{in}}(S) > \sum_{i \in S} k_i^{\text{out}}(S)\ \forall S \in C\right\}\right|}{|C|}
\end{aligned} \tag{7}
$$

where S is a community in the partition C, $k_i^{in}(S)$ is the number of edges connecting node i to other nodes in S, and $k_i^{\text{out}}(c)$ is the number of edges connecting node i to the nodes outside S. The larger the value means the better partition. Moreover, a strong community is also a weak community, whereas the reverse is not correct. Thus, *weakRatio*(C) is always equal or greater than *strRatio*(C). Because the four EA-based algorithms (i.e., GACD, MOCD, MOGA-Net, and NSGA-Net) have close NMI results, we further compare the community structure they discover with the *ratio of strong (or weak) communities*. The results are demonstrated in Figures 8(c) and (d). It shows that NSGA-Net has the highest *ratio of strong (and weak) communities* in most conditions. Although NSGA-Net is not always the best one for small μ, it consistently achieves the best performance when μ is large. We know that, with the increment of μ, the community structure becomes fuzzier, and it is more difficult to identify the community structure. The results imply that NSGA-Net has better potential to discover fuzzy community structures compared with other algorithms.

5.4. Efficiency Experiments

To observe the time efficiency of NSGA-Net, we record the running time of all algorithms on the aforementioned artificial network experiments. Figure 9(a) shows the results. We can find that the aggregation-based algorithms (e.g., GN and GN Fast) are



(a) Running time of different algorithms

(b) Running time of NSGA-Net on different parameters

FIGURE 9. Time efficiency of NSGA-Net.

more efficient. Because of population evolutionary, the EA-based algorithms (e.g., GACD, MOCD, NSGA-Net, MOGA-Net) cost more running time. Moreover, the running time of GACD is smaller than other EA based algorithms, because only one single objective is optimized in GACD. The results illustrate that multiobjective community discovery algorithms usually need longer running time, although they can discover more accurate community structures.

NSGA-Net is based on NSGA-II that has the time complexity $\mathcal{O}(gs^2)$ (Deb et al. 2002). Thus, the time complexity of NSGA-Net is $\mathcal{O}(gs^2 f(O))$ (g is the running generation and s is the population size. $f(O)$ is the complexity of calculating the objective function O). It shows that the running generation and population size greatly affect the time efficiency of NSGA-Net. We further do experiments to validate the time efficiency of NSGA-Net on different parameter settings. Figure 9(b) demonstrates the running time of NSGA-Net on the aforementioned artificial networks when it is set with different population sizes and running generations. The results clearly show that the running time of NSGA-Net almost linearly increases with the running generation. With the increment of population size, the running time of NSGA-Net also increases. The experiments confirm the time complexity analysis of NSGA-Net.

Generally, the larger running generation and population size in NSGA-Net will lead to the better algorithm performance. However, it also needs more running time. Therefore, we can trade off the effectiveness and efficiency of NSGA-Net by setting the proper parameters according to applications. For small-scale networks, we can set the larger running generation and population size for more accurate results. For large-scale networks, we can set the not large running generation and population size for acceptable results in not long time.

5.5. Discussions

Note that the same optimizer is used for all the objectives, and the same number of individuals is evaluated in the two EA-based algorithms: NSGA-Net and GACD. We believe that the multiobjective optimization in NSGA-Net contributes to its performance improvement. In the MOP framework, the characteristics of community structure are measured from different angles, which reduces the risk that one single objective may have bias on a certain kind of network. Moreover, the population evolution in multiobjective optimization process trade-offs the balance of the multiple objectives; thus, it helps to avoid local optima.

Why are only the negatively correlated objectives suitable for the multiobjective community detection method? We think that the negatively correlated objectives have the opposite effects on the number of communities, which can make the number of communities dynamic. It can avoid the algorithm converges to trivial solutions. In addition, the negatively correlated objectives also reflect different aspects of communities, and they can potentially enhance the diversity of solutions. It helps to avoid prematurity. The positively correlated objectives are equivalent to a single objective, and thus the multiobjective community detection becomes a single-objective community detection in fact. As for the case with independent objectives, where the conflict among objectives is not strong, the optimizer is hard to effectively explore the objective space to avoid the local optimal. Therefore, the multiobjective community detection only optimizing over negatively correlated objectives, rather than positively correlated or independent objectives, can effectively improve the accuracy of community partition.

6. RELATED WORK

In the past decade, many community detection algorithms have been proposed, most of which are based on the optimization over a single objective function. These single-objective community detection algorithms are usually implemented in two ways. Some algorithms consider community detection as an optimization problem that is solved through optimizing an evaluation criterion, such as the Q criterion (Newman and Girvan 2004) in modularity optimization methods (Guimera and Amaral 2005; Pizzuti 2008; Shi et al. 2010b), the "cut" function (Kannan et al. 2004) in the spectral method (Pothen et al. 1990). Some algorithms design heuristic rules to detect community structures. Such examples include the edge betweenness (Newman and Girvan 2004) and link clustering coefficient (Radicchi et al. 2004). Different from these single-objective algorithms, our NSGA-Net simultaneously optimizes multiple objectives.

Recently, the multiobjective optimization technique has widely been applied in data mining problems. For example, Handle and Knowles (2007) apply EMO to boost clustering performance. The multiobjective-based neural network ensemble is also applied to improve multilabel classification performances (Shi et al. 2011a). Particularly, more and more researchers have been aware that the community detection is a MOP in nature (Brandes, Delling, and Gaetler 2008; Fortunato and Barthelemy 2007; Martin and Carl 2007) and several multiobjective community detection algorithms have been proposed. Pizzuti proposed the MOGA-Net (Pizzuti 2009), which simultaneously optimizes the *CommunityScore* and *CommunityFitness* with NSGA-II. Shi et al. proposed the MOCD (Shi et al. 2012), in which the optimization objectives are two components of modularity Q and the optimization framework is PESA-II (Corne et al. 2001). Agrawal (2011) designed BOCD to maximize modularity Q and *Community Score* simultaneously. Moreover, Folino and Pizzuti (2010) proposed a multiobjective evolutionary community detection for dynamic networks, in which the algorithm optimizes the accuracy and smoothness of community structures. Distinct from these approaches, NSGA-Net is a general multiobjective community detection algorithm, which can optimize any objectives. More importantly, this paper deeply exploits the characteristics and relations of objective functions and analyzes the effect of relations of optimization objectives on the performance of multiobjective community detection.

In our previous work, we analyzed the structural characteristics of communities identified by objective functions in the single-objective setting (Shi et al. 2010a) and preliminarily studied the selection of objective functions in multiobjective community detection (Shi et al. 2011b). On the basis of these work, this paper has the following two significant contributions. (1) It greatly extends these existing work. We formally proposed the objective selection problem in the multiobjective community detection paradigm and analyzed its importance. After analyzing the characteristics and correlations of objective functions, we do a large number of experiments to test the performance of our proposed multiobjective algorithm NSGA-Net under different objective combination and draw the conclusion that NSGA-Net with negatively correlated objectives usually leads to a better performance. (2) It does extensive experiments to validate our conclusion. Except the experiments on artificial networks, we validate our conclusion on 12 real networks. We design new criteria to evaluate the comparison results and compare NSGA-Net with more algorithms.

7. CONCLUSION

In this paper, we study an important issue in the multiobjective community detection: what type of objectives should be optimized? After proposing a general multiobjective community detection solution NSGA-Net, we first systematically analyze the characteristics of communities identified by 11 objectives and reveal their intrinsic correlations (i.e., positively correlated, independent, and negatively correlated). Then, we compare the performances of NSGA-Net optimizing over different types of objectives to those of a single-objective-based approach optimizing over the original single objective. The extensive experiments show that the multiobjective community detection does not necessarily improve the accuracy of community partition, and its performance largely depends on the selection of objective functions. NSGA-Net only with a pair of negatively correlated objectives remarkably improves the performance. Moreover, NSGA-Net with a pair of positively correlated or independent objectives has no obvious lifts on performance. With a pair of negatively correlated objectives, NSGA-Net also performs better than other popular community detection algorithms.

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